Digital Signal Processing

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Q1
Use the following trigonometric identity to derive an expression for \( \cos 8\theta \) in terms of \( \cos 9\theta, \cos 7\theta, \) and \( \cos \theta \).

\[
\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y
\]

Q2
Plot the graph of the following function:

\[
s(t) = \begin{cases} 
2t & 0 \leq t \leq \frac{1}{2} \\
\frac{1}{2}(4-2t) & \frac{1}{2} \leq t \leq 2 \\
0 & \text{elsewhere}
\end{cases}
\]

A1

\[
\begin{align*}
\cos 9\theta &= \cos(3\theta + 6\theta) = \cos 3\theta \cos 6\theta - \sin 3\theta \sin 6\theta \\
\cos 7\theta &= \cos(3\theta - 2\theta) = \cos 3\theta \cos 2\theta + \sin 3\theta \sin 2\theta \\
\cos 9\theta + \cos 7\theta &= 2 \cos 8\theta \cos \theta \\
\Rightarrow \cos 8\theta &= \frac{\cos 9\theta + \cos 7\theta}{2 \cos \theta}
\end{align*}
\]

A2

A3

Q3
Considering the function given in question 2, derive an equation for \( x(t) = s(t-2) \) and plot the graph of the function.
A3

$$x(t) = s(t - 2) = \begin{cases} 
2(t-2) & 2 \leq t \leq 2.5 \\
(8 - 2t) & 2.5 \leq t \leq 4 \\
0 & \text{elsewhere}
\end{cases}$$

Q4

Considering the function given in question 2, derive an equation for $$x_2(t) = s(t+1)$$ and plot the graph of the function.

A4

$$x(t) = s(t + 1) = \begin{cases} 
2(t+2) & -1 \leq t \leq -\frac{1}{2} \\
(2-2t) & -\frac{1}{2} \leq t \leq 1 \\
0 & \text{elsewhere}
\end{cases}$$

Q5

Derive an equation for the following graph of the function.

A5

$$x(t) = \begin{cases} 
(t+1) & -1 \leq t \leq 0 \\
(1- \frac{1}{2}t) & 0 \leq t \leq 2 \\
0 & \text{elsewhere}
\end{cases}$$

Q6

In the following waveform, it is possible to measure both a positive and a negative value of $$t_1$$ and then calculate the corresponding phase shifts. Which phase shift is within the range $$-\pi < \phi \leq \pi$$? Verify that the two phase shifts differ by $$2\pi$$.
For the following signal, \( x(t) = 20\cos(2\pi(40)t - 0.4\pi) \), find \( G \) and \( t_1 \) so that the signal \( y(t) = Gx(t - t_1) \) is equal to \( 5\cos(2\pi(40)t) \); i.e., obtain an expression for \( y(t) = 5\cos(2\pi(40)t) \) in terms of \( x(t) \).

Demonstrate that expanding the real part of \( \exp(j(\alpha + \beta)) \) will lead to the following identity.

\[
\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta
\]

Also show that the following identity is obtained from the imaginary part.

\[
\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta
\]

Show that the following representation can be derived for the real sine signal:

\[
A \sin(\alpha t + \phi) = \frac{1}{2} X e^{-j\phi} e^{-j\alpha t} + \frac{1}{2} X^* e^{j\phi} e^{j\alpha t}
\]

where \( X = Ae^{j\phi} \). In this case, the interpretation is that the sine signal is also composed of two complex exponentials with the same positive and negative frequencies, but the complex coefficients multiplying the terms are different from those of the cosine signal. Specifically, the sine signal requires additional phase shifts of \( \pm \pi/2 \) applied to the complex amplitude \( X \) and \( X^* \), respectively.
\( \frac{1}{2} A e^{i \omega_0 t} e^{-j \phi} + \frac{1}{2} A e^{-i \omega_0 t} e^{j \phi} \) \\
\( = \frac{1}{2} A (e^{i(\omega t + \phi)} + e^{-i(\omega t + \phi)}) \)

\[ \cos(\omega t + \phi) \]

Recall trig identity:

\[ \cos(\phi) + \sin(\phi) \]

\[ A \sin(\omega t + \phi) \]

Use \( A \cos(\theta) = A \cos \theta \cos(\alpha t) - A \sin \theta \sin(\alpha t) \) to show that the sum

\[ 1.7 \cos(20\pi t - 70\pi/180) + 1.9 \cos(20\pi t + 200\pi/180) \]

reduces to \( A \cos(20\pi t + \phi) \), where

\[ A = 1.532 \]

\[ \phi = 2.475 \text{ rads} \]

Consider the two sinusoids,

\[ x_1(t) = 5 \cos(2\pi(100)t + \pi/3) \]

\[ x_2(t) = 4 \cos(2\pi(100)t - \pi/4) \]

Obtain the phasor representations of these two signals, add the phasors, plot the two phasors and their sum in the complex plane, and show that the sum of the two signals is

\[ x_3(t) = 5.536 \cos(2\pi(100)t + 0.2747) \]

In degrees the phase should be 157.4. Examine the plots in Fig. 2-16 to see whether you can identify the cosine waves

• \( x_1 \), \( x_2 \), and \( x_3 \) D \( x_1 \) \( x_2 \) \( x_3 \) \[ X = e^{j\omega_0 t} \] and \( z'(t) = X' e^{j\omega_0 t} \)

Determine the value of \( \omega_0 \) for which the differential equation is satisfied.
FIRST DERIVATIVE:
\[ \frac{d}{dt} \dot{x}(t) = j\omega \dot{x} e^{j\omega t} \]
\[ \frac{d}{dt} \ddot{x}(t) = -j\omega \left( \frac{K}{m} \right) \dot{x} e^{j\omega t} \]

2ND DERIVATIVE:
\[ \frac{d}{dt} \dot{x}(t) = -j\omega \frac{K}{m} \dot{x} e^{j\omega t} \]
\[ \frac{d}{dt} \ddot{x}(t) = -j\omega \left( \frac{K}{m} \right) \dot{x} e^{j\omega t} \]

If \(-\omega^2 = -\frac{K}{m}\), then the differential equation is satisfied \(\Rightarrow k_b = \frac{K}{m}\).