Digital Signal Processing

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Digital Filtering of Analog Signals

Lecture 13

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READING ASSIGNMENTS

• This Lecture:
  – Chapter 6, Sections 6-6, 6-7 & 6-8

• Other Reading:
  – Recitation: Chapter 6
    • FREQUENCY RESPONSE EXAMPLES
  – Next Lecture: Chapter 7

LECTURE OBJECTIVES

• Two Domains: Time & Frequency
• Track the spectrum of $x[n]$ thru an FIR Filter: Sinusoid-IN gives Sinusoid-OUT
• UNIFICATION: How does Frequency Response affect $x(t)$ to produce $y(t)$?

TIME & FREQUENCY

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k] x[n-k]$$

THE DIFFERENCE EQUATION IN TIME DOMAIN

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k}$$

$$H(e^{j\hat{\omega}}) = h[0] + h[1] e^{-j\hat{\omega}} + h[2] e^{-2j\hat{\omega}} + h[3] e^{-3j\hat{\omega}} + \ldots$$

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Ex: DELAY by 2 SYSTEM

Find $h[n]$ and $H(e^{j\omega})$ for $y[n] = x[n-2]$

$$x[n] \xrightarrow{h[n]} y[n] \quad b_k = \{ 0, 0, 1 \}$$

$$h[n] = \delta[n-2]$$

$$H(e^{j\omega}) = \sum_{k=0}^{M} \delta[k-2]e^{-j\omega k}$$

GENERAL DELAY PROPERTY

Find $h[n]$ and $H(e^{j\phi})$ for $y[n] = x[n-n_\delta]$

$$h[n] = \delta[n-n_\delta]$$

$$H(e^{j\phi}) = \sum_{k=0}^{M} \delta[k-n_\delta]e^{-j\phi k} = e^{-j\phi n_\delta}$$

ONLY ONE non-ZERO TERM for $k$ at $k = n_\delta$

FREQ DOMAIN --> TIME ??

• START with $H(e^{j\phi})$ and find $h[n]$ or $b_k$

$$H(e^{j\phi}) = 7e^{-j2\phi} \cos(\phi)$$

PREVIOUS LECTURE REVIEW

• SINUSOIDAL INPUT SIGNAL
  – OUTPUT has SAME FREQUENCY
  – DIFFERENT Amplitude and Phase

• FREQUENCY RESPONSE of FIR
  – MAGNITUDE vs. Frequency
  – PHASE vs. Freq
  – PLOTTING

$$H(e^{j\phi}) = |H(e^{j\phi})|e^{j\angle H(e^{j\phi})}$$
FREQ. RESPONSE PLOTS

• DENSE GRID \((\omega)\) from \(-\pi\) to \(+\pi\)
  \(-\omega = -\pi: (\pi/100): \pi;\)
• \(HH = \text{freqz}(bb,1,\omega)\)
  - VECTOR \(bb\) contains Filter Coefficients
  - DSP-First: \(HH = \text{freqz}(bb,1,\omega)\)

\[H(e^{j\omega}) = \sum_{k=0}^{M} b_k e^{-j\omega k}\]

EXAMPLE 6.2

Find \(y[n]\) when \(H(e^{j\omega})\) is known and \(x[n] = 2e^{j\pi/4} e^{j(\pi/3)n}\)

\[x[n] \quad H(e^{j\omega}) \quad y[n] \]

\[H(e^{j\omega}) = (2 + 2 \cos \omega)e^{-j\omega}\]

EXAMPLE 6.2 (answer)

Find \(y[n]\) when \(x[n] = 2e^{j\pi/4} e^{j(\pi/3)n}\)

One Step - evaluate \(H(e^{j\omega})\) at \(\omega = \pi/3\)

\(H(e^{j\omega}) = (2 + 2 \cos \omega)e^{-j\omega}\)

\(H(e^{j\omega}) = 3e^{-j\pi/3} \quad \text{at } \omega = \pi/3\)

\(y[n] = \left(3e^{-j\pi/3}\right) \times 2e^{j\pi/4} e^{j(\pi/3)n} - 6e^{-j\pi/12} e^{j(\pi/3)n}\)

EXAMPLE: COSINE INPUT

Find \(y[n]\) when \(H(e^{j\omega})\) is known and \(x[n] = 2\cos(\frac{\pi}{3} n + \frac{\pi}{4})\)

\[x[n] \quad H(e^{j\omega}) \quad y[n] \]

\(H(e^{j\omega}) = (2 + 2 \cos \omega)e^{-j\omega}\)

EX: COSINE INPUT (ans-1)

Find \(y[n]\) when \(x[n] = 2\cos(\frac{\pi}{3} n + \frac{\pi}{4})\)

\[2\cos(\frac{\pi}{3} n + \frac{\pi}{4}) = e^{j(\pi n/3 + \pi/4)} + e^{-j(\pi n/3 + \pi/4)}\]

\[\Rightarrow x[n] = x_1[n] + x_2[n]\]

\(y_1[n] = H(e^{j\pi/3}) e^{j(\pi n/3 + \pi/4)}\)

\(y_2[n] = H(e^{-j\pi/3}) e^{-j(\pi n/3 + \pi/4)}\)

\(\Rightarrow y[n] = y_1[n] + y_2[n]\)
EX: COSINE INPUT (ans-2)

Find $y[n]$ when $x[n] = 2 \cos\left(\frac{\pi}{4} n + \frac{\pi}{4}\right)$

$$H(e^{j\hat{\omega}}) = (2 + 2 \cos \hat{\omega}) e^{-j\hat{\omega}}$$

$$y_1[n] = H(e^{j\pi/4}) e^{j\pi/3} = 3 e^{-j\pi/3} e^{j\pi/4}$$

$$y_2[n] = H(e^{-j\pi/4}) e^{-j\pi/3} = 3 e^{j\pi/3} e^{-j\pi/4}$$

$$y[n] = 3 e^{j(\pi/12)} + 3 e^{-j(\pi/12)}$$

$\Rightarrow y[n] = 6 \cos\left(\frac{\pi}{6} n - \frac{\pi}{12}\right)$

SINUSOID thru FIR

- IF $H'(e^{j\hat{\omega}}) = H(e^{-j\hat{\omega}})$
- Multiply the Magnitudes
- Add the Phases

$$x[n] = A \cos(\hat{\omega} n + \phi)$$

$$\Rightarrow y[n] = A H(e^{j\hat{\omega}}) \cos(\hat{\omega} n + \phi + \angle H(e^{j\hat{\omega}}))$$

LTI Demo with Sinusoids

DIGITAL “FILTERING”

- SPECTRUM of $x(t)$ (SUM of SINUSOIDS)
- SPECTRUM of $x[n]$
- Is ALIASING a PROBLEM?
- SPECTRUM $y[n]$ (FIR Gain or Nulls)
- Then, OUTPUT $y(t) = SUM$ of SINUSOIDS

FREQUENCY SCALING

- TIME SAMPLING:
  - IF NO ALIASING:
  - FREQUENCY SCALING

$$t = n T_s$$

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

11-pt AVERAGER Example

$$H(e^{j\hat{\omega}}) = \frac{\sin\left(\frac{11}{2} \hat{\omega}\right)}{11 \sin\left(\frac{1}{2} \hat{\omega}\right)} e^{-j5\hat{\omega}}$$

$$x(t) = \cos(2\pi(25)t) + \cos(2\pi(250)t - \frac{\pi}{2})$$
D-A FREQUENCY SCALING

\[ x(t) \xrightarrow{A-to-D} x[n] \xrightarrow{H(e^{j\omega})} y[n] \xrightarrow{D-to-A} y(t) \]

- **TIME SAMPLING:** \( t = nT_s \Rightarrow n \leftarrow tf_s \)
- **RECONSTRUCT up to 0.5f_s**
  - FREQUENCY SCALING

\[ \omega = \hat{\omega} f_s \]

TRACK the FREQUENCIES

\[ x(t) \xrightarrow{A-to-D} x[n] \xrightarrow{H(e^{j\omega})} y[n] \xrightarrow{D-to-A} y(t) \]

- 250 Hz • 0.5\( \pi \)
- 25 Hz • 0.05\( \pi \)
- 25 Hz • 0.05\( \pi \)
- 250 Hz

\( f_s = 1000 \text{ Hz} \)

NO new freqs

11-pt AVERAGER

EVALUATE Freq. Response

\[ H(e^{j\hat{\omega}}) = \frac{\sin(\frac{1}{11} \hat{\omega})}{11 \sin(\frac{1}{11} \hat{\omega})} e^{-j5\hat{\omega}} \]

At \( \hat{\omega} = 0.5\pi \)

\[ H(e^{j\hat{\omega}}) = \frac{\sin(\frac{1}{11} (0.5\pi))}{11 \sin(\frac{1}{11} (0.5\pi))} e^{-j5(0.5\pi)} \]

\[ = \frac{\sin(2.75\pi)}{11 \sin(0.25\pi)} e^{-j2.5\pi} \]

\[ = 0.0909 e^{-j0.5\pi} \]

EVALUATE Freq. Response

\[ x(t) = \cos(2\pi(25)t) + \sin(2\pi(250)t) \]
evaluating at 25 and 250 Hz.

\[ H(e^{j2\pi(25)/1000}) = \frac{\sin(\pi(25)(11)/1000)}{11 \sin(\pi(25)/1000)} e^{-j2\pi(25)/1000} \]

\[ H(e^{j2\pi(250)/1000}) = \frac{\sin(\pi(250)(11)/1000)}{11 \sin(\pi(250)/1000)} e^{-j2\pi(250)/1000} \]

\[ y(t) = 0.8811 \cos(2\pi(25)t - \pi/2) + 0.0909 \sin(2\pi(250)t - \pi/2) \]

EFFECTIVE RESPONSE

DIGITAL FILTER

LOW-PASS FILTER

Equivalent Continuous Time Frequency Response for \( f_s = 1000 \)
FILTER TYPES

- LOW-PASS FILTER (LPF)
  - BLURRING
  - ATTENUATES HIGH FREQUENCIES
- HIGH-PASS FILTER (HPF)
  - SHARPENING for IMAGES
  - BOOSTS THE HIGHS
  - REMOVES DC
- BAND-PASS FILTER (BPF)

B & W IMAGE

B&W IMAGE with COSINE

FILTERED B&W IMAGE

ROW of B&W IMAGE

FILTERED ROW of IMAGE

11-Point Averaging: 5-Sample Delay Equalization
EFFECTIVE Freq. Response

- Assume NO Aliasing, then
  - ANALOG FREQ <-> DIGITAL FREQ
  - So, we can plot:
  - Scaled Freq. Axis

\[ \hat{\omega} = \omega T_s = \frac{\omega}{f_s} \]

\[ H(\omega T_s) \text{ vs. } \omega \]

TIME & FREQ DOMAINS

- LTI: Linear & Time-Invariant
  - COMPLETELY CHARACTERIZED by:
    - IMPULSE RESPONSE \[ h[n] \] (time domain)
    - FREQUENCY RESPONSE

\[ x[n] \rightarrow H(e^{j\omega}) \rightarrow y[n] \]

- Two DOMAINS: time & frequency
  - Go back and forth QUICKLY

SINUSOID thru FIR

\[ x[n] = x_0 + \sum_{k=1}^{N} \left( \frac{x_1}{2} e^{j\omega k} + \frac{x_2}{2} e^{-j\omega k} \right) \]

\[ = x_0 + \sum_{k=1}^{N} \left( X_1 \cos(\omega k\Delta t) + X_2 \right) \]

\[ y[n] = H(\omega) x[n] \]

If \( H(-\hat{\omega}) = H^*(\hat{\omega}) \), the corresponding output is:

\[ y[n] = H(\hat{\omega}) x[n] = \sum_{k=1}^{N} \left( H(\hat{\omega}) \frac{x_1}{2} e^{j\omega k} + \frac{x_2}{2} e^{-j\omega k} \right) \]

\[ = H(\hat{\omega}) x_0 + \sum_{k=1}^{N} \left( H(\hat{\omega}) X_1 \cos(\omega k\Delta t) + X_2 \right) \]