Q1. Mark the following statements as (T)rue or (F)alse.

a. When a signal is sampled, the information in between samples is lost. (T)
b. An image is a two dimensional signal. (T)
c. Aliasing happens when the sampling frequency is greater than twice the signal frequency. (F)
d. A moving average filter is a low-pass filter which smooths the signal. (F)
e. Digital systems cannot be easily duplicated.

Q2. We want to measure temperature variation in the classroom over the time digitally (by using a computer). Briefly explain (by using block diagrams if necessary to describe each steps) how to implement the temperature measurement system (how to represent temperature variation in computers).

Temperature Sensor \(\rightarrow\) Amplifier \(\rightarrow\) Analog Signal Processing \(\rightarrow\) ADC \(\rightarrow\) Digital Data

ADC Involve:
1. Anti-aliasing Filtering
2. Sampling
3. Quantification
4. Binary coding

Q3. a. Accuracy of an ADC (the analog voltage represented by the LSB) is given to be ±0.0625 V. If the input range of the ADC is 0 to 16 volts, what is the number of bits in this ADC?

\[
0.0625 = \frac{16 - 0}{2^n} \implies 2^n = \frac{16}{0.0625} = 256 = 2^8
\]

\[n = 8\text{ bits.}\]
b. The same ADC is used to record a speech signal for 2 minutes. If the sampling frequency of the ADC is 20 kHz (k samples/second), calculate the required memory space (in terms of the byte) to store the speech signal.

required memory space = 2 \times 60 \times 20,000 = 24,000,000 bytes

Q4. Frequency spectrum of a signal is given as the following:

a. Write an equation for the signal \( x(t) \) defined by this frequency spectrum.

\[ x(t) = 2 + 4 \cos(2 \pi 30t) + 2 \cos(2 \pi 130t) \]
\[ = 2 + 4 \cos(60\pi t) + 2 \cos(260\pi t) \]

b. Write \( x[n] \) after the signal digitized by an ADC with a sampling frequency of 100 Hz.

\[ X[n] = X[nT_s] = 2 + 4 \cos(60\pi n T_s) + 2 \cos(260\pi n T_s) \]
\[ T_s = \frac{1}{f_s} = \frac{1}{100} \]
\[ X[n] = 2 + 4 \cos\left(\frac{60\pi n}{100}\right) + 2 \cos\left(\frac{260\pi n}{100}\right) \]
\[ X[nT_s] = 2 + 4 \cos(0.6\pi n) + 2 \cos(2.6\pi n) \]
\[ = 2 + 4 \cos(0.3 \times 2\pi n) + 2 \cos(1.3 \times 2\pi n) \]

There is an aliasing in (b)? If there is prove it.

\[ X[n] = 2 + 4 \cos(0.6\pi n) + 2 \cos((2 + 0.6)\pi n) \]
\[ = 2 + 4 \cos(0.6\pi n) + 2 \cos(2\pi n + 0.6\pi n) \]
\[ = 2 + 4 \cos(0.6\pi n) + 2 \cos(2\pi n) \cos(0.6\pi n) - 2 \sin(2\pi n) \sin(0.6\pi n) \]
\[ = 2 + 4 \cos(0.6\pi n) + 2 \cos(0.6\pi n) = 2 + 6 \cos(0.6\pi n) \]

So, there is an aliasing.
d. What happens to the signal \( x(t) \) when the sampled signal in (b) is reconstructed?

\[
x(t) = x \left[ \frac{t}{T_s} \right] = 2 + 6 \cos \left( \frac{0.67 \pi t}{0.01} \right) = 2 + 6 \cos (60 \pi t)
\]

\[
x(t) = 2 + \cos (2 \pi 30 t)
\]

Because of aliasing, signal with 130 Hz is lost. It appears as a 30 Hz signal. Therefore, magnitude of 30 Hz signal became 6.

**Q5.** Find the three important parameters: amplitude \( A \), phase \( \phi \) and fundamental angular frequency \( \omega_0 \) which define a particular sinusoid for the two signals on the following graph. Take time delays approximately.

**Signal 1:**

\[
A = 1.5, \quad T = 1.25 s, \quad \omega_0 = \frac{2 \pi}{T} = 1.6 \pi \text{ rad/s}
\]

\[
\phi = 0.2 \pi = -\omega_0 t, \quad t = -0.25 s
\]

**Signal 2:**

\[
A = 2, \quad T = 1.6 s, \quad \omega_0 = \frac{2 \pi}{T} = 1.25 \pi \text{ rad/s}
\]

\[
\phi = 0.2 \pi = -\omega_0 t, \quad t = -0.2 s
\]

**Q6.** A periodic signal is defined by the equation

\[
x(t) = 2 + 4 \cos \left( 40 \pi t - \frac{1}{5} \pi \right) + 3 \sin \left( 60 \pi t \right) + 4 \cos \left( 120 \pi t - \frac{1}{3} \pi \right)
\]

a. Determine the fundamental frequency \( \omega_0 \), the fundamental period \( T_0 \), and coefficients \( a_k \) in the Fourier representation for that signal.

\[
x(t) = 2 + \frac{3}{2} e^{-j (4 \pi t - \frac{1}{5} \pi)} + \frac{3}{2} e^{j (4 \pi t - \frac{1}{5} \pi)} + \frac{1}{2} e^{-j (6 \pi t)} + \frac{1}{2} e^{j (6 \pi t)} + \frac{1}{2} e^{-j (12 \pi t - \frac{1}{3} \pi)} + \frac{1}{2} e^{j (12 \pi t - \frac{1}{3} \pi)}
\]

\[
x(t) = 2 + 2 e^{-j \frac{2}{5} \pi} + 2 e^{j \frac{2}{5} \pi} + 1.5 e^{-j \frac{3}{2} \pi} + 1.5 e^{j \frac{3}{2} \pi} + 1.5 e^{-j \frac{5}{2} \pi} + 1.5 e^{j \frac{5}{2} \pi} + 1.5 e^{-j \frac{7}{2} \pi} + 1.5 e^{j \frac{7}{2} \pi} + 1.5 e^{-j \frac{9}{2} \pi} + 1.5 e^{j \frac{9}{2} \pi}
\]

\[
\text{Ref: } T = \frac{1}{f} = \frac{1}{2} = e^{j \frac{3}{2} \pi} \rightarrow \frac{1}{2} = e^{j \frac{3}{2} \pi}
\]

\[
t_1 = 0.1 \text{ s}, \quad t_2 = 0.01 \text{ s}, \quad t_3 = 0.01 \text{ s}, \quad \therefore T_0 = \frac{1}{0.01} = 100 \text{ Hz}
\]

\[
\omega_0 = 20 \pi \text{ rad/s}
\]
Q7. Consider the following periodic function $x(t)$.

a. Find the Fourier coefficients $a_k$, for $k \neq 0$ in the Fourier series representation of $x(t)$. (10)

$$a_k = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-j \left( \frac{2\pi}{T_0} \right) kt} dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} e^{-j \left( \frac{2\pi}{T_0} \right) kt} dt$$

$$= \frac{1}{T_0} e^{-j \left( \frac{\pi}{kT_0} \right)t} \left[ \frac{e^{j \left( \frac{\pi}{kT_0} \right)t}}{j \frac{\pi}{kT_0}} \right]_{-T_0/2}^{T_0/2}$$

$$a_k = \frac{e^{j \frac{\pi k}{2}} - e^{-j \frac{\pi k}{2}}}{j \frac{\pi k}{2}} = \frac{\sin \frac{\pi k}{2}}{j \frac{\pi k}{2}}$$

b. Determine the DC coefficient of the Fourier series, $a_0$. (05)

$$a_0 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) dt = \frac{1}{T_0} \left[ \frac{T_0}{u} t \right]_{-T_0/2}^{T_0/2}$$

$$a_0 = \frac{1}{T_0} \left( \frac{T_0}{u} - \left( \frac{-T_0}{u} \right) \right) = \frac{1}{T_0} \cdot \frac{T_0}{u} = \frac{1}{2}$$