Reconstruction of nonuniformly sampled time-limited signals using prolate spheroidal wave functions

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Abstract

Shannon's sampling theory is based on the reconstruction of bandlimited signals which requires infinite number of uniform time samples. Indeed, one can only have finite number of samples for numerical implementation. In this paper, as a dual of the bandlimited reconstruction, a solution for time-limited signal reconstruction from nonuniform samples is proposed. The system model we present is based on the idea that time-limited signals which are also nearly bandlimited can be well approximated by a low-dimensional subspace. This can be done by using prolate spheroidal wave functions as the basis. The order of the projection on this basis is obtained by means of the time–frequency dimension of the signal, especially in the case of non-stationary signals. The reconstruction requires the estimation of the nonuniform sampling times by means of an annihilating filter. We obtain the reconstruction parameters by solving a linear system of equations and show that our finite-dimensional model is not ill-conditioned. The practical aspects of our method including the dimensionality reduction are demonstrated by processing synthetic as well as real signals.

1. Introduction

The Shannon's sampling theory (also attributed to E.T. Whittaker and V.A. Kotel’nikov) [1,2] is based on uniform sampling of bandlimited signals. If a finite energy signal $x(t)$ has a Fourier transform $X(\omega)$ and is bandlimited, that is

$$x(t) \in B = \{x(t) \in L^2(\mathbb{R}) : X(\omega) = 0, |\omega| > \Omega_{\text{max}}\},$$

then $x(t)$ can be reconstructed from uniform samples $\{x(kT_s)\}$:

$$x(t) = \sum_k x(kT_s) \frac{\sin(\Omega_{\text{max}}(t - kT_s)/\pi)}{\Omega_{\text{max}}(t - kT_s)/\pi},$$

where $T_s = \pi/\Omega_{\text{max}}$ is the sampling period. Three issues of practical interest arise:

- Dimensionality of the signal space. The problem with this interpolation is with the infinite support of the basis and with the degrees of freedom of the signal space.
- Nonuniform sampling. The implementation of uniform sampling is not realistic [3–5].
- Finite-domain implementation. Numerical reconstruction of sampled signals approximates the signal in a finite domain while the theoretical results are derived in an infinite-dimensional space [6].

From the Landau–Pollak–Slepian dimensionality theorem [7,8] a discrete signal with a bandwidth $\beta$ and $N$ samples in a finite interval, has at most $N/\beta$ degrees of freedom. Dimensionality reduction in the reconstruction of signals...