A Novel Visualization Technique in Bond-Graph Method for Modeling of a Generalized Stewart Platform

İbrahim Yıldız, Vasfi Emre Ömürlü and Ahmet Sağırlı
School of Mechanical Engineering
Yıldız Technical University
İstanbul, Beşiktaş 34349, Türkiye
{iyildiz, omurlu & sagirli}@yildiz.edu.tr

Abstract - This paper represents dynamic modeling of generalized stewart platform manipulator by Bond Graph method with a new spatial visualization method and the state-space representation of the dynamic equations of the system. Dynamic model includes all the dynamics and gravity effects, linear motor dynamics as well as the viscous friction at the joints. Following modeling of actuation system and of main structure, unification of these two is accomplished. Linear DC motors are utilized and modelled as the actuation system. Since overall system consists of high nonlinearity originated from geometric nonlinearity and gyroscopic forces, resultant derivative causality problem caused by rigidly coupled inertia elements is addressed and consequent system state-space equations are presented.

Index Terms – Stewart Platform, Bond Graph, System Dynamics, Spatial Kinematics, State-Space

I. INTRODUCTION

The dynamic models of parallel mechanisms are more complicated than serial mechanisms because of their closed-form structure. Nevertheless, dynamic models are important for the control of parallel mechanisms. The stewart platform is a six-degree-of-freedom mechanism with two plates mounted by six linearly actuated legs and first proposed as a flight simulator in 1965 by D. Stewart [1]. Many applications related to physical simulation of spatial motion have been put into realization. Present study mentions obtaining dynamic formulation of a generalized stewart platform using bond graph method with a new spatial visualization technique.

Bond-Graph method has been established as a new approach to model, analyze and control various dynamical systems by Prof. Henry Paynter in 1961, in which energy bonds are utilized to define the energy flow between the physical elements of any system. Since it is a graphical approach and resultant dynamic equations are in state-space form, the method is straightforward and ideal for following control design problems. Many researchers have worked on Bond-Graph modeling of mechanical systems. Reference [2,3] developed Bond-Graph techniques for hydraulic, mechatronic and thermodynamic systems. Reference [4] studied a comprehensive research on Bond-Graph techniques whose results are applicable to the robotics and spatially moving objects.

Reference [5] developed a Bond-Graph technique obtaining Lagrange and Hamilton functions by modeling systems with high degree of nonlinearity. The method is based on defining the dependent velocities of the system with general coordinates and local coordinates. Reference [6] studied on a systematic modeling method of spatially moving mechanisms with Bond-Graph technique. Reference [7] worked on modeling mechanical systems with vector Bond-Graph method. Reference [8] used Lagrange and Hamilton equations for solving the derivative causality problem on geometrically nonlinear systems.

The inverse dynamics of the stewart platform mechanism is solved using Newton-Euler method by Reference [9], in which force-moment equalities are used to reach the transmitted forces through the joints and the actuators, assuming symmetrical and thin legs and frictionless joints. Reference [10] solved the inverse dynamics of a system assuming symmetrical and thin legs and frictionless joints.

Reference [11] developed motion equations for a system using Lagrangian equations which utilizes kinetic-potential energy equations to find the velocity and acceleration characteristics of any dynamic system. A similar equation system is developed by Reference [12] for control purposes.

Reference [13] applied kinematic vector Bond-Graph method to a telescopic rotary crane, obtained state space formulation of this mechanism and solved derivative causality problem by reducing the inertial elements to the independent velocity ports using virtual inertia and a gyristor element.

Reference [14,15] used linear graph technique to generate the system equations of a stewart platform for impedance control, yet, the details of the approach and the developed mathematical model were not provided.

In this study, Bond-Graph modeling of a stewart platform mechanism is addressed including all dynamical and gravitational effects. The nonlinear model includes linear motor dynamics and all joints have viscous friction. The inverse kinematics equations are used for modeling of the mechanism.

II. MATHEMATICAL MODELING OF A STEWART PLATFORM MECHANISM

A. Assumptions and Presentation of the Stewart Platform Mechanism
The physical model of a generalized Stewart platform mechanism is shown in Fig. 1.

Fig. 1 The physical model of a generalized Stewart platform mechanism

The modeled mechanism has six linear motors as actuators. Each actuator is attached to the upper platform and the lower platform by spherical and universal joints, respectively. Upper part of the Stewart Platform Mechanism performs following motions:

Linear motion along \( x_p, y_p \) and \( z_p \) axis; \( t_x, t_y, t_z \) respectively. Rotation around \( x_p, y_p \) and \( z_p \) axis; \( \gamma, \theta, \psi \) respectively. It is assumed that:

- Center of gravity of upper platform is located at the origin of the mobile frame.
- Center of gravity of actuators is located about a distance of the 2/3 of the total actuator length from the lower connection point.
- External force and moment effects are neglected.
- Bearing frictions and gravitational forces are present on the system.

In Fig. 1, upper platform with the attached \( \{P\} \) coordinate system is performing linear and angular motion with linear and angular velocities, \( \dot{t} \) and \( \dot{\omega} \), respectively. The attached linear actuators are performing both linear and angular velocities, \( \dot{L} \) and \( \dot{\theta} \), respectively. The parallel interaction between actuators causes angular velocities on each leg.

### B. Kinematics

Position of upper and lower connection points can be expressed with the following equations,

\[ \begin{align*}
\dot{P} &= \begin{bmatrix} r_p \cos(\lambda_i) & r_p \sin(\lambda_i) & 0 \end{bmatrix} \\
\dot{B} &= \begin{bmatrix} r_i \cos(\Lambda_i) & r_i \sin(\Lambda_i) & 0 \end{bmatrix}
\end{align*} \]  

(1)

(2)

Equation (1) denotes the position vector of upper connection points with respect to frame \( \{P\} \). \( r_p \) and \( \lambda_i \) shows radius of upper platform and the angle between upper connection points and \( z \) axis of upper platform’s coordinate system, respectively. Equation (2) represents the position vector of lower platform connection points with respect to base frame \( \{B\} \). \( r_B \) and \( \Lambda_i \) shows radius of lower platform and the angle between lower connection points and \( x \) axis of lower platform’s coordinate system, respectively. Upper connection points can be expressed with respect to the base frame by multiplying (1) with the rotation matrix,

\[ R = \begin{bmatrix}
\cos(\theta \psi) & \cos(\gamma \psi) - \sin(\gamma \psi) \sin(\theta \psi) & \sin(\gamma \psi) - \cos(\gamma \psi) \sin(\theta \psi) \\
\cos(\theta \gamma) - \sin(\theta \gamma) \sin(\psi \gamma) & \cos(\gamma \psi) & \sin(\gamma \psi) \sin(\theta \gamma) - \cos(\gamma \psi) \cos(\theta \gamma)
\end{bmatrix} \]  

(3)

Where \( c = \cos \) and \( s = \sin \) are functions.

\[ \ddot{q}_i = R \ddot{p}_i \]  

(4)

From (3) and (4), leg vectors and leg lengths can be expressed by the terms of upper platforms position variables.

\[ \ddot{S}_i = \ddot{q}_i + \ddot{t} - \ddot{b}_i \]  

(5)

\[ L_i = [\ddot{S}_i] \]  

(6)

Unit vectors along the legs can be expressed by dividing leg vector by the leg length.

\[ \ddot{S} = \ddot{S} / L \]  

(7)

Linear velocity along the legs can be defined as,

\[ \dot{L} = \ddot{S} \]  

(8)

where \( \ddot{S} \) is the velocity of upper connection points and it can be expressed as,

\[ \ddot{S} = \omega \times \ddot{q} + \ddot{t} \]  

Also, angular velocity of the actuators can be expressed as,

\[ \dot{\theta}_i = \left( \dot{s}_i \times \ddot{S}_i \right) / L_i \]  

(9)

(10)

### III. KINEMATIC BOND GRAPH MODEL OF A STEWART PLATFORM MECHANISM

Translational and rotational velocities along the actuators can be expressed by the terms of upper platform velocities.

\[ \dot{L} = \begin{bmatrix} s_{q_x} - s_{q_w} & s_{q_y} - s_{q_w} & s_{q_z} - s_{q_w} & s_{q_z} & s_{q_y} & s_{q_x} \end{bmatrix} \]  

(11)

\[ \dot{\theta}_i = \begin{bmatrix} s_{q_x} + s_{q_w} & -s_{q_y} & -s_{q_z} & 0 & -s_{q_z} & s_{q_y} \end{bmatrix} \]  

(12)

Equations (11) and (12) can be shown as,

\[ \begin{bmatrix} \dot{\omega} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} K_{1,3} \begin{bmatrix} \dot{L} \\ \dot{\theta} \end{bmatrix} \\ K_{4,6} \begin{bmatrix} \dot{L} \\ \dot{\theta} \end{bmatrix} \end{bmatrix} \]  

(13)

\[ \begin{bmatrix} \dot{\omega} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} K_{1,3}^\theta \begin{bmatrix} \dot{L} \\ \dot{\theta} \end{bmatrix} \\ K_{4,6}^\dot{} \begin{bmatrix} \dot{L} \\ \dot{\theta} \end{bmatrix} \end{bmatrix} \]  

(14)
In (13) and (14), the angular velocity of upper platform multiplies by the first, second and third columns of $K^L$ and $K^θ$, same as the linear velocity of upper platform multiplies by the fourth, fifth and sixth columns of $K^L$ and $K^θ$. Consequently, $K_{1,3}$ signifies the effects of angular velocity of upper platform to the leg velocities and also, $K_{4,6}$ represents the effects of linear velocity of upper platform to the leg velocities.

Kinematic vectorial bond graph model of the mechanism for one leg which is obtained from (13) and (14) is given in Fig. 2. In complete model, each parallel energy ports have 12 energy bonds and each leg has different translational and rotational velocities. A new model is developed to understand the relationship between the kinematic structure of the system and the physical model. Fig. 2 shows two main velocity variables of each leg, ($L_i, \theta_i$), which can be expressed with same energy bonds by using new kinematic approach.

Assuming two distinct energy ports “0” and “1”, two vectorial effort variables are acting on these energy ports, Fig. 3.

Left side of Fig. 3 represents common vector Bond-Graph of two distinct energy bonds. In the right side of Fig. 3, energy bonds and energy ports are associated by one energy bond and new complex vector Bond-Graph is established. The right side of Fig. 3 represents two distinct energy ports and two dependent energy ports. “1.0” port shows one “1” and one “0” distinct energy ports. The energy associated with “a” flows from “0” port to “1” port as in the energy related to the “b” flows from the other “0” port to the other “1” port.

If two distinct energy ports bounded each other by one energy port, this case can be shown as in Fig. 4.

Right side of Fig. 4 illustrates developed complex vector Bond-Graph model and the one on the left side represents using classical approach.

Now, it is possible to show overall kinematic model of Stewart platform mechanism by using complex kinematic vector Bond-Graph method, Fig. 5.

**IV. DYNAMIC BOND GRAPH MODEL OF A STEWART PLATFORM MECHANISM**

Attaching the physical elements of the system to the kinematic model, dynamic Bond-Graph model of Stewart platform mechanism can be obtained. Vectorial dynamic Bond-Graph model of the mechanism for one leg is shown in Fig. 6.

**A. Derivative Causality Solution**

When two or more inertial elements are attached to “1” port, system state variables become dependent and this causes derivative causality problem. This means, independent
velocities of these elements becomes dependent velocities when inertial elements attached and this causes derivative causality problem on the system. This problem results in an implicit differential equation and may cause numerical problems in solution. This problem can be solved by transforming inertial elements of dependent velocities to the ports of independent velocities. Therefore, inertial elements should be reduced to the ports of independent velocities as a virtual inertia matrix and a gyristor element. Dependent velocities, independent velocities and inertial velocities of the system should be defined before reducing the system. Translational and rotational velocity of legs can be chosen as dependent velocities, therefore upper platform’s translational and rotational velocities can be chosen as independent velocities. All dependent and independent velocities can be chosen as inertial velocities. Dependent velocities can be expressed as,

\[
\dot{q}_b = \begin{bmatrix} \dot{L}_1 & \dot{\theta}_1 & L_2 & \dot{L}_3 & \dot{L}_5 & L_4 & \dot{\theta}_4 & L_5 & \dot{\theta}_5 & L_6 & \dot{\theta}_6 \end{bmatrix}^T
\]  

(15)

In (15), \( \dot{\theta}_i \) is 3 element matrix. Therefore, \( \dot{q}_b \) is a 24 element matrix. Independent velocities can be defined as,

\[
\dot{q}_i = \begin{bmatrix} \omega_i & \omega_i & \omega_i & \dot{\theta}_i & \dot{\theta}_i & \dot{\theta}_i \end{bmatrix}^T
\]  

(16)

Inertial velocities can be expressed as,

\[
\dot{q}_i = \begin{bmatrix} \dot{q}_b \end{bmatrix}
\]  

(17)

The relationship between independent and inertial velocities is \( T_I \) matrix and can be defined as,

\[
\dot{q}_i = T_I \cdot \dot{q}_i
\]  

(18)

All inertial elements of the system should be defined with a single inertia matrix before obtaining virtual inertial matrix and gyristor element. Inertial elements should be arranged using inertial velocity matrix. The virtual inertial matrix and the gyristor element can be expressed as,

\[
[I] = [T_I]^T \cdot [I] \cdot [T_I]
\]  

(19)

and

\[
[G] = [T_I]^T \cdot [I] \cdot \begin{bmatrix} \dot{q}_i \end{bmatrix}
\]  

(20)

The derivative of relation matrix \( T_I \) is needed to calculate the gyristor element. It is useful way to derivate this matrix during simulation of complete model of the system.

After reducing the inertial elements to two ports, bond graph of the system is shown in Fig. 7.

Fig. 7 shows the complete Bond-Graph model of mechanical part of the system. The gyroscopic forces acting on the platform is expressed by one modulated gyrator element, Fig.8. The inertial elements which are shown in Fig. 8 are defined in the virtual inertial matrix.

In Fig. 8, flow equations can be expressed as,

\[
M_x = H_x \omega_x - H_y \omega_y
\]

(21)

\[
M_y = -H_x \omega_y + H_y \omega_y
\]

(22)

\[
M_z = -H_x \omega_z + H_y \omega_z
\]

(23)

where, \( H_x = I_x \omega_x, \), \( H_y = I_x \omega_y, H_z = I_z \omega_z, \).

B. Modeling the Actuation Mechanism (Linear DC Motor)

The actuation of the system is based on linear DC actuators which are chosen as legs of the Stewart Platform Mechanism. Each linear motor is mounted to mobile platform by spherical joints and is mounted to the stationary platform by universal joints. Complete Bond-Graph model of the mechanism for one leg is shown in Fig. 9 after adding the actuation part.

The gravitational effect on legs and upper platform are modeled by effort sources, and the actuation inputs of the legs are modeled by effort sources in Fig. 9. Right side of the picture shows the linear motion and the left side shows angular motion of the leg.

V. SYSTEM STATE-SPACE EQUATIONS

State equations can be expressed as,
\[
\frac{d}{dt}\begin{bmatrix} \omega \\ I \end{bmatrix} = \begin{bmatrix} I \end{bmatrix}^T \begin{bmatrix} M_{i_1} & M_{i_1} & M_{i_1} & F_{i_1} & F_{i_1} & F_{i_1} \end{bmatrix}^T,
\]

(24)

\[
\frac{d}{dt}(i_{a_i}) = L_{a_i} V_{a_i}, (i = 1..6)
\]

The constitutive law of the gyrator element can be expressed as,

\[
\begin{bmatrix} M_{Gx} & M_{Gy} & M_{Gz} & F_{Gx} & F_{Gy} & F_{Gz} \end{bmatrix} =
\begin{bmatrix} \omega_x & \omega_y & \omega_z & t_x & t_y & t_z \end{bmatrix}^T
\]

(25)

The right side of the equation (24) is created with the terms of dependent variables which must be expressed with the terms of state variables to find state-space equations. From the parallel junctions of the mechanical part of the Bond-Graph model in Fig. 9, inner equations are obtained as,

\[
\tilde{M}_f = \sum_{i=1}^{6} \tilde{M}_{i} - \sum_{i=1}^{6} \tilde{M}_{i}^\theta - \tilde{M}_G - \tilde{M}_\theta
\]

(26)

\[
\tilde{F}_f = \sum_{i=1}^{6} \tilde{F}_{i} - \sum_{i=1}^{6} \tilde{F}_{i}^\theta - \tilde{F}_G - m_p \theta
\]

(27)

In equation (26) and (27), \(M_G\) and \(F_G\) terms are effort variables of the gyrator element which are illustrated in Fig. 8. \(\tilde{M}_{i}^\theta\) and \(\tilde{F}_{i}^\theta\) terms are moments and forces that are generated by the actuators, respectively. The reaction moment and the reaction force of upper platform acting on legs are indicated by \(\tilde{M}_G\) and \(\tilde{F}_G\), respectively. In equation (26), \(\tilde{M}_\theta\) shows the gyroscopical moments of upper platform. Open form of equation (26) can be expressed as,

\[
\dot{M}_{i_1} = \sum_{i=1}^{6} \left( K_{11} \dot{F}_{i_1} - \sum_{i=1}^{6} \left[ \begin{array}{c} \frac{K_{11}}{K_{12}} \\ \frac{K_{11}}{K_{13}} \\ \frac{K_{11}}{K_{14}} \end{array} \right] \right) \begin{bmatrix} M_{i_1}^\theta \\ M_{i_1}^\theta \\ M_{i_1}^\theta \end{bmatrix} - G_{1.6} [\omega]_t
\]

(28)

\[
-(H_x \omega_y - H_y \omega_x)
\]

\[
\dot{M}_{i_2} = \sum_{i=1}^{6} \left( K_{12} \dot{F}_{i_2} - \sum_{i=1}^{6} \left[ \begin{array}{c} \frac{K_{22}}{K_{23}} \\ \frac{K_{22}}{K_{24}} \end{array} \right] \right) \begin{bmatrix} M_{i_2}^\theta \\ M_{i_2}^\theta \end{bmatrix} - G_{2.6} [\omega]_t
\]

(29)

\[
-(H_y \omega_x + H_x \omega_y)
\]

\[
\dot{M}_{i_3} = \sum_{i=1}^{6} \left( K_{13} \dot{F}_{i_3} - \sum_{i=1}^{6} \left[ \begin{array}{c} \frac{K_{33}}{K_{34}} \end{array} \right] \right) \begin{bmatrix} M_{i_3}^\theta \end{bmatrix} - G_{3.6} [\omega]_t
\]

(30)

and open form of equation (27) also expressed as,

\[
F_{i_1} = \sum_{i=1}^{6} \left( K_{11} \dot{F}_{i_1} - \sum_{i=1}^{6} \left[ \begin{array}{c} \frac{K_{11}}{K_{12}} \\ \frac{K_{11}}{K_{13}} \\ \frac{K_{11}}{K_{14}} \end{array} \right] \right) \begin{bmatrix} M_{i_1}^\theta \\ M_{i_1}^\theta \\ M_{i_1}^\theta \end{bmatrix} - G_{4.6} [\omega]_t
\]

(31)

\[
F_{i_2} = \sum_{i=1}^{6} \left( K_{12} \dot{F}_{i_2} - \sum_{i=1}^{6} \left[ \begin{array}{c} \frac{K_{22}}{K_{23}} \\ \frac{K_{22}}{K_{24}} \end{array} \right] \right) \begin{bmatrix} M_{i_2}^\theta \\ M_{i_2}^\theta \end{bmatrix} - G_{5.6} [\omega]_t
\]

(32)

\[
F_{i_3} = \sum_{i=1}^{6} \left( K_{13} \dot{F}_{i_3} - \sum_{i=1}^{6} \left[ \begin{array}{c} \frac{K_{33}}{K_{34}} \end{array} \right] \right) \begin{bmatrix} M_{i_3}^\theta \end{bmatrix} - G_{6.6} [\omega]_t
\]

(33)

The \(G_{i.6}\) term in equation (28) shows the first row of gyrator matrix. Leg forces (\(F_{i_1}\)) and leg moments (\(M_{i_1}^\theta\)) are unknown variables of these equations. These unknown variables can be found by writing the constitutive law for “0” energy ports. All unknown variables should be expressed by the terms of state variables. Leg moments can be given as,

\[
M_{i_1}^\theta = R_{\theta} \dot{\theta}_{i_1} = R_{\theta} K_{1.6}^{\theta} [\omega]_t (i = 1..6)
\]

(34)

\[
M_{i_2}^\theta = R_{\theta} \dot{\theta}_{i_2} = R_{\theta} K_{2.6}^{\theta} [\omega]_t (i = 1..6)
\]

(35)

\[
M_{i_3}^\theta = R_{\theta} \dot{\theta}_{i_3} - m_n g \frac{2L}{3} \cos(\theta_{i_3}) =
\]

(36)

\[
R_{\theta} K_{3.6}^{\theta} [\omega]_t - m_n g \frac{2L}{3} \cos(\theta_{i_3}) (i = 1..6)
\]

(37)

Leg forces can also be given as,

\[
F_{i_1} = F_{s_1} - R_{L} K_{1.6}^{L} [\omega]_t - m_n g \sin(\theta_{i_1})
\]

(38)

Actuating force of the leg (\(F_{s_1}\)) is also an unknown variable in equation (37) and can be expressed by using the Bond-Graph model in Fig. 8.

\[
V_{s_1} = V_{c_1} - V_{s_1} - V_{R_1} (i = 1..6)
\]

(39)

where, \(V_{s_1} = L_i K_i = K_{1} [(\omega) \cdot i F_{1}] K_i, F_{a(i)} = K_i a_i, V_{R_1} = i_n R_a\).

By writing the \(V_{s_1}, F_{s_1}, V_{R_1}\) in equation (38), acting forces can be expressed as,

\[
V_{s_1} = V_{c_1} - K_{1.6}^{L} [\omega]_t K_i - i_n R_a (i = 1..6)
\]

(40)

From equations (26), (27), (38), \(M_{i_1}, M_{i_2}, M_{i_3}, F_{i_1}, F_{i_2}, F_{i_3}, F_{s_1}, V_{s_1}, V_{R_1}\) terms can be expressed by the terms of state variables.

\[
M_{i_1} = \omega_x E_{m_1}^{m_1} + \omega_y E_{m_1}^{m_1} + \omega_z E_{m_1}^{m_1} + i_x E_{m_1}^{m_1} + \nonumber
\]

\[
i_y E_{m_1}^{m_1} + t_z E_{m_1}^{m_1} + \sum_{n=1}^{6} \left( i_{a_n} E_{m_1}^{m_1} + T_{m_1}^{m_1} + T_{m_2}^{m_1} \right)
\]

(41)
\[ M_{f_1} = \omega_1 E_{\omega_1}^{m_1} + \omega_2 E_{\omega_2}^{m_2} + \omega_3 E_{\omega_3}^{m_3} + t_1 E_{t_1}^{m_1} + t_2 E_{t_2}^{m_2} + \sum_{n=1}^{6} i_{n} E_{n_1}^{m_1} + T_{a_1}^{m_1} + T_{\theta_1}^{m_1} \]

\[ M_{f_2} = \omega_1 E_{\omega_1}^{m_1} + \omega_2 E_{\omega_2}^{m_2} + \omega_3 E_{\omega_3}^{m_3} + t_1 E_{t_1}^{m_1} + t_2 E_{t_2}^{m_2} + \sum_{n=1}^{6} i_{n} E_{n_1}^{m_1} + T_{a_1}^{m_1} + T_{\theta_1}^{m_1} \]

\[ F_{a_1} = \omega_1 E_{\omega_1}^{F_1} + \omega_2 E_{\omega_2}^{F_2} + \omega_3 E_{\omega_3}^{F_3} + t_1 E_{t_1}^{F_1} + \sum_{n=1}^{6} i_{n} E_{n_1}^{F_1} + T_{a_1}^{F_1} + T_{\theta_1}^{F_1} \]

\[ F_{a_2} = \omega_1 E_{\omega_1}^{F_1} + \omega_2 E_{\omega_2}^{F_2} + \omega_3 E_{\omega_3}^{F_3} + t_1 E_{t_1}^{F_1} + t_2 E_{t_2}^{F_1} + \sum_{n=1}^{6} i_{n} E_{n_1}^{F_1} + T_{a_1}^{F_1} + T_{\theta_1}^{F_1} \]

\[ V_{a_1} = \omega_1 E_{\omega_1}^{V_1} + \omega_2 E_{\omega_2}^{V_2} + \omega_3 E_{\omega_3}^{V_3} + t_1 E_{t_1}^{V_1} + t_2 E_{t_2}^{V_1} + \sum_{n=1}^{6} i_{n} E_{n_1}^{V_1} + T_{a_1}^{V_1} + T_{\theta_1}^{V_1} \]

Equations (40), (41), (42), (43), (44), (45), and (46) should be plug into equation (24).

These equation series can be expressed by matrix norm as,

\[
\frac{d}{dt} \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 & t_1 & t_2 & i_{a_1} & i_{a_2} & i_{a_3} & i_{a_4} & i_{a_5} & i_{\theta_1} \\ \end{bmatrix}^T = \begin{bmatrix} A_{6x6} & B_{6x6} \\ C_{6x6} & D_{6x6} \end{bmatrix} \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 & t_1 & t_2 & i_{a_1} & i_{a_2} & i_{a_3} & i_{a_4} & i_{a_5} & i_{\theta_1} \\ \end{bmatrix}^T + \begin{bmatrix} f_{6x6} \end{bmatrix} + \begin{bmatrix} \int f_{1x6} \end{bmatrix} U_{2x6}
\]

Equation (47) represents the state-space equation for a Stewart Platform Mechanism [16].

VI. CONCLUSION

Dynamic characteristics of mechanical systems take important place for efficient control. Parallel mechanisms, especially Stewart platform mechanisms, have complex system dynamics and are completely nonlinear. Therefore, in the present study, kinematics and dynamics of a general Stewart platform mechanism are obtained by Bond-Graph method using a novel visualization technique and nonlinear state-space representation of the system dynamics is developed. An important problematic point of obtaining dynamic equations is the possible derivative causality problem. If there is a disregarded derivative causality problem during calculations, numerical problems and errors occur when simulating the system. Thus, transforming elements of dependent velocities to the ports of independent velocities, derivative causality problem is solved. At the end of the study, a state-space model of the mechanism is provided.

ACKNOWLEDGMENT

This work is supported by The Scientific and Technological Research Council of Turkey (TUBITAK) grant 3501-105M192.

REFERENCES