Directional Wavelet Transform in the Context of Complex Quadrature Doppler Signals

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***Abstract*—Most Doppler ultrasound systems employ quadrature demodulation techniques at the detection stage. The information concerning flow direction encoded in the phase relationship between in-phase and quadrature phase channels is not obvious at this stage. A method based on the utilization of complex wavelets and negative scales has been described. It eliminates the intermediate processing stages by mapping directional information in the scale domain.**

***Index Terms*—Complex wavelets, Doppler, wavelet transform.**

I. INTRODUCTION

MOST Doppler ultrasound systems employ quadrature demodulation techniques at the detection stage [1]. The incoming RF signal from an ultrasonic transducer is multiplied by a 90° phase-shifted version of the transmitted signal as well as the transmitted signal. After low pass filtering the HF components, in-phase and quadrature phase components of the audio Doppler signal are obtained. The information concerning flow direction, which is encoded in the phase relationship between in-phase and quadrature phase channels is not obvious at this stage. The complex fast Fourier transform (FFT) can be used to map directional information in the frequency domain [2]. The FFT results are displayed as a sonogram, which is a form of the time frequency representation of Doppler ultrasound signals. The Fourier transform (FT) expands a time domain signal into a family of waves, which are completely unlocalized in time. Although it assumes that the signal is stationary, in practice, most natural signals are nonstationary. Hence, the windowed Fourier transform (WFT) has been used widely to observe evolution of the signal both in time and frequency. However, it has an inherent time frequency resolution limitation. Using longer frame sizes may cause transient behavior of the signal to be missed.

In this context, the wavelet transform (WT) may be used to study Doppler ultrasound signals, especially those containing short lived unidirectional transients caused by emboli, which are particles larger than red blood cells found within the circulatory system [3]. The WT has also been used successfully to analyze, model, and compute turbulent flows [4]. It allows the time-frequency resolution compromise to be optimized [5]. Similar to the complex FT, the WT provides directional information in the scale domain within the context of complex quadrature signals. This may be accomplished, in part, by extending the WT analysis to negative scales.

II. WAVELET TRANSFORM AND TIME SCALE ANALYSIS

The WT is performed by projecting a signal $s(t)$ onto a family of zero-mean functions deduced from an elementary function $\psi(t)$ by translations and dilations. It is given by

$$W_s(a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{+\infty} s(t)\psi^* \left(\frac{t-b}{a}\right) dt \qquad (1)$$

The variables a and b control the scale and position of the wavelet, respectively. The WT is a linear transformation and covariant under translations and dilations. It creates a two-dimensional (2-D) representation of a one-dimensional signal, with the horizontal axis as time and vertical axis as scale. The third dimension is the amplitude of the WT coefficients. This allows exact localization of any abrupt change, or an exact time and duration of a short signal, which may not be evidenced by conventional signal processing techniques.

A. Complex Signals and Wavelets

In many applications, we are required to work with complex signals, as it allows the representation of a modulated band limited signal as its baseband equivalent. The FT coefficients of such signals are no longer symmetric. Likewise, it is convenient to work with analytic wavelets, which can separate amplitude and phase components and allow the measurement of the time evolution of frequency transients [6]. In the context of Doppler ultrasound, an analytic signal is a subset of the complex quadrature signals, in which both positive and negative frequency components have a physical meaning. For the FT, the complex exponential ensures the separation of flow direction in the frequency domain. The positive frequency spectrum represents the signals resulting from the forward flow and the negative frequency spectrum represents the signals resulting from the reverse flow. By analogy, a similar property can be attained by the use of complex wavelets. The WT for processing quadrature Doppler signals can be implemented in such a way that only the coefficients resulting from the forward flow components are obtained when the scale is positive, and only the coefficients resulting from the reverse flow components are obtained when the scale is negative. This is attained by the sine-cosine formulation, which naturally exists in some common wavelets such as the Morlet wavelet [7], which is obtained by taking a complex sine wave and localizing it with a Gaussian envelope. Ignoring

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Manuscript received May 26, 2000. This work was supported by Project Grant PG99064, British Heart Foundation. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. R. Shenoy.

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Publisher Item Identifier S 1070-9908(00)08358-9.

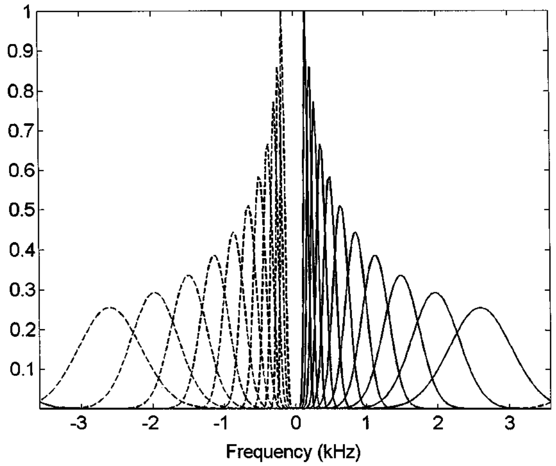

Fig. 1. Frequency spectra of the scale dependent complex Morlet wavelet for 22 different scales ($-11 < a < 11$). Continuous plot is the spectra of upper analytic wavelet (positive scales) and the dashed plot lower analytic wavelet (negative scales).

the translation parameter b, the scale dependent Morlet wavelet is given by

$$\psi(t/a) = \pi^{-1/4} e^{i\omega_0 t/a} e^{-t^2/2a^2} \tag{2}$$

where ω_0 is the nondimensional frequency and usually assumed to be 5 to 6 to satisfy the admissibility condition. The FT of (2) is given by

$$\Psi(\omega) = \begin{cases} \sqrt{2}\pi^{1/4}|a|e^{-(a\omega-\omega_0)^2/2}H(\omega), & \text{if } a > 0 \\ \sqrt{2}\pi^{1/4}|a|e^{-(a\omega+\omega_0)^2/2}H(-\omega), & \text{if } a < 0. \end{cases} \tag{3}$$

where H stands for the Heaviside step function. From (3), one can observe that a frequency spectrum of an upper analytic signal is obtained for $a > 0$ and a frequency spectrum of a lower analytic signal is obtained for $a < 0$. Fig. 1 shows the frequency spectra of the Morlet wavelet for a range of negative and positive scales.

B. Method and Applications

The WT of a signal $s(t)$ with the Morlet wavelet is given by

$$W_s(a,b) = \frac{\pi^{-\frac{1}{4}}}{\sqrt{|a|}} \int_{-\infty}^{+\infty} s(t) e^{i\omega_0\left(\frac{t-b}{a}\right)} e^{-0.5\left(\frac{t-b}{a}\right)^2}\, dt. \tag{4}$$

If the number of scales is J, a complete set of directional wavelet coefficients can be mapped over the scales from $a = -J$ to $a = J$, excluding $a = 0$. In order to see how negative scales have been utilized to obtain directional wavelet coefficients, let us evaluate (4) for one positive and one negative scale. Assuming $b = 0$ for simplicity, the WT of the signal $s(t)$ for $a = 1$ and $a = -1$ are given, respectively, by

$$W_s(1,0) = C \int_{-\infty}^{+\infty} s(t) e^{i\omega_0 t} e^{-0.5t^2}\, dt \tag{5a}$$

$$W_s(-1,0) = C \int_{-\infty}^{+\infty} s(t) e^{-i\omega_0 t} e^{-0.5t^2}\, dt \tag{5b}$$

where the constant $C = \pi^{-1/4}|a|^{-1/2}$. Since the WT is equivalent to the convolution of the wavelet function and the signal under investigation, it can be easily implemented using the fast convolution, which corresponds to taking the FT of the wavelet function and the signal independently and multiplying them in the frequency domain. Taking the FT of (5a) and (5b) yields, respectively

$$F\{W_s(1,0)\} = CS(\omega)e^{-0.5(\omega-\omega_0)^2}H(\omega) \tag{6a}$$

$$F\{W_s(-1,0)\} = CS(\omega)e^{-0.5(\omega+\omega_0)^2}H(-\omega) \tag{6b}$$

where $S(\omega)$ is the FT of the signal $s(t)$ and C is a constant. The inverse FT of (6a) and (6b) yields complex wavelet coefficients formed only by the forward and the reverse flow components respectively.

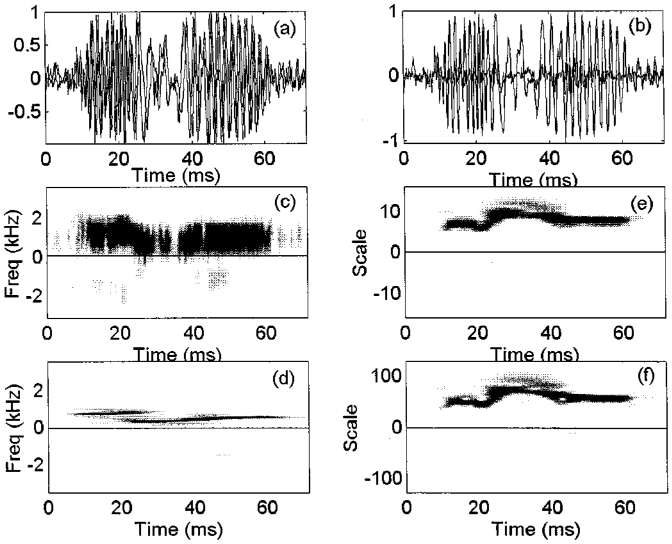

Fig. 2. (a) Embolic quadrature Doppler signal and (b) corresponding directional signals. Respective time frequency plots with (c) 16 point and (d) 256 point Gaussian windows and time scale plots with (e) 16 scales and (f) 128 scales Morlet wavelet.

A long quadrature embolic Doppler signal recorded from a patient with symptomatic carotid stenosis by a transcranial Doppler system, respective directional signals obtained using the Hilbert transform process [2], and its time frequency and time-scale plots are illustrated in Fig. 2. Fig. 3 shows a directional low intensity embolic signal corrupted by a bidirectional low frequency artifact and respective 3-D time frequency and time scale plots. The sampling frequency for both signals was 7150 Hz. A Gaussian window with the highest overlap ratio was used for the time-frequency analysis. For the time scale analysis, the complex Morlet wavelet was used. Although the relationship between the equivalent Fourier period and the wavelet scale can be derived analytically for a particular wavelet function, the convention is to use the time scale representation.

III. RESULTS AND CONCLUSION

The time frequency result with the WFT is a function of the analysis window size and the nonstationary signal duration. Despite having good temporal resolution, the frequency resolution is poor with 16 point WFT for the signal given in Fig. 2(a) and (c). On the other hand, the frequency resolution is good and temporal resolution is poor with 256 point WFT [Fig. 2(d)]. Unlike

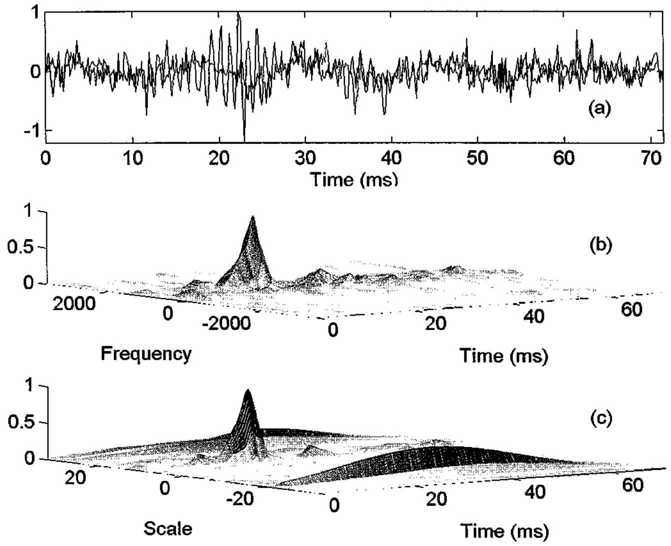

Fig. 3. (a) Forward and reverse signals of a low intensity short embolic Doppler signal corrupted by a low frequency bidirectional artifact, (b) corresponding three-dimensional (3-D) time frequency plot with 32 point Gaussian window, and (c) time scale plot with 32 scales Morlet wavelet.

the WFT, the WT analysis provides good frequency resolution for the low frequency signals and good time resolution for the high frequency signals and hence an optimized time frequency localization for the analysis of complex quadrature Doppler signals. The WT with 16 scales [Fig. 2(e)] and 128 scales [Fig. 2(f)] provide almost the same time scale representation. In Fig. 3, bidirectional low frequency signal components caused by an artifact are almost lost in the time frequency plot due to poor frequency resolution [Fig. 3(b)]. On the other hand, they are clearly visible in the 3-D time scale plot with a comparable description of the short embolic signal [Fig. 3(c)]. This is important to discriminate low intensity embolic signals from large artifacts.

The method described above utilizes complex wavelets and negative scales in a physical context for mapping directional wavelet coefficients in the scale domain. The positive scales will produce the appropriate wavelet coefficients when processing the signals with positive frequency spectrum. On the other hand, negative scales will produce the wavelet coefficients caused by the signal components having only negative frequency spectrum. The method also eliminates the intermediate processing stages such as FIR Hilbert transform and associated delay filtering for obtaining directional Doppler signals for the time scale analysis.

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