Gain-Sensitivity Analysis for Cascaded Two-Ports
and Application to Distributed-Parameter Amplifiers

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ABSTRACT: Enhancement of a gain-sensitivity analysis of electrical networks is presented by computing gain sensitivities with respect to network parameters. A simple and versatile method. The so-called chain-sensitivity matrix is presented and compared with the current method in the literature, gain factorization, for the gain sensitivities of the cascaded networks. Analytical formulas are derived to calculate gain sensitivities of the T and II types of distributed-parameter amplifiers with respect to the physical length / and characteristic impedance $Z_0$, rather than using a time-intensive computer-based perturbation method. The numerical results of the T- and II-type amplifiers for the design targets of noise figure $F_{req} = 0.46$ dB (⇔ 1, 12) input VSWR $V_{req} = 1$, power gain $G_{req} = 12$ dB (⇔ 15, 86) and the bandwidth $B = 2$ GHz – 11 GHz are given in comparison to each other. © 2004 Wiley Periodicals, Inc. Int J RF and Microwave CAE 14: 462–474, 2004.

Keywords: sensitivity; transducer gain; chain parameters; matching circuit; characteristic impedance; phase constant

I. INTRODUCTION

Sensitivities, or partial derivatives of certain network functions with respect to circuit parameters, are very important in the design of microwave circuits. An aspect of the circuit design is the optimization of the electrical and noise performances. Efficient gradient-based optimization requires calculation of the circuit sensitivities. The study of the network sensitivity and its automated calculation was developed at least as early 1972 by Calahan [1]. Since then, sensitivity calculation by the adjoin method and its variants, as well as direct computation of network partial derivatives, has become textbook material [2, 3]. Nevertheless, until now, there have been a very limited number of considerable works on the analytical expressions for the sensitivities of the performance-measure functions of microwave networks in the literature [4–7].

In [4], analytical formulas of the gradients were supplied to the optimization of a distributed amplifier with T-type matching circuits. In [4], the analysis the gradient of the gain was worked out by the so-called gain factorization method (utilized in our work as well), in which gradients of the other performance-measure functions, such as noise, input VSWR, and output VSWR, are expressed in terms of gain gradients with the proper terminations. In [5], noise figure and spot-noise parameters sensitivities were derived analytically by using a nodal approach for both single and cascaded two-port networks, and an application was derived for a two-stage amplifier with common-gate active matching. In addition, all the results were verified by the numerical-perturbation method. In [6], excellent information of the sensitivity analysis was gathered for the microwave circuits.

In our work, an enhancement of the gain-sensitivity analysis for the cascaded two-ports is presented by using the two different abovementioned approaches: gain factorization and chain-sensitivity matrix. These
sensitivities can be related to any passive or active parameter of the element in any position of the network. Rather than employing the adjoin method, which is one of the major techniques with regard to sensitivity in linear-circuit theory and can be applied to any type of circuit configuration, a simple and easy method known as chain sensitivity is established for formulation of sensitivity, which straightforwardly uses the configuration knowledge of the original network and results in weighting coefficients that are dependent upon the position of the two-port of the element. This type of sensitivity formulation was applied to many microwave circuits with distributed elements. In particular, after solving the problem of definite target space [8], a cascaded configuration is very easy to use in the design with together the Darlington representation of the source and load impedances [9]. Designing a microwave amplifier using the potential-noise, input-VSWR, and gain performances, with predetermined bandwidth for the employed active device, can be considered as a significant development in current state-of-the-art microwave technology. The motivation underlying this work is essentially to investigate how and in what specific ways the sensitivities of the distributed parameters effect the circuit performance. This can provide the answer to the question of the tolerance limitations of the technology utilized in the realization process. At the same time, an equally important application area is the fact that analytical gradient expressions obtained by an easy and simple method can be used in the optimization procedure of microwave amplifiers. In the CAD of microwave circuits, direct expressions for the gain sensitivities can improve the speed of computations, and also safeguard against possible numerical difficulties associated with the perturbation method.

In the realization and production stages, the network and system parameters usually change; the effect of these changes upon the system performance can be analyzed by using the system sensitivities.

Real networks and systems must usually operate under changing environmental conditions (for example, at changing temperature, humidity, atmospheric pressure, electromagnetic radiation, and so forth). Circuit elements change their values due to natural aging. By taking into account these dependencies, we can determine network or system sensitivities with respect to environmental-condition parameters.

Information about network sensitivities may be used extensively in computer-aided optimal-tolerance assignment, optimal centering, and postproduction tuning.

**II. SENSITIVITY**

In this work, sensitivity is defined as the partial derivative of a derivable function \( F \) with respect to the \( x \) parameter:

\[
D_x(F) = \frac{\partial F}{\partial x}.
\]

The common normalized sensitivity can be defined in the form of either

\[
S_x(F) = \frac{\partial (\ln F)}{\partial (\ln x)} = \frac{\partial_x(F)}{F} \quad (2a)
\]

or

\[
S_x(F) = \frac{\partial F}{\partial x} = \frac{x}{F} D_x(F). \quad (2b)
\]

From the definition in eq. (2b), the relative change in \( F \) can be related to its cause \( \partial x / x \) as follows:

\[
\frac{\partial F}{F} = S_x(F) \frac{\partial x}{x}. \quad (2c)
\]

So in order that the \( F \) function will not be sensitive to \( x \), the limits of the \( S_x(F) \) function should be

\[
-1 \leq S_x(F) \leq 1. \quad (2d)
\]

For a general case, \( F \) and \( x \) may be vectors consisting of \( m \) real-circuit functions and \( k \) real parameters, respectively:

\[
F(x) = [F_1(x), F_2(x), \ldots, F_m(x)]^T, \quad (3)
\]

\[
x = [x_1, x_2, \ldots, x_k]^T. \quad (4)
\]

Here, the \( x \)-circuit parameter vector can be combined as follows:
Where \( x^o, 1 \times k \) is a dimensioned nominal parameter vector and \( \Delta x \) is a differential vector with the same dimensions:

\[
x^o = [x_1^o, x_2^o, \ldots, x_k^o]^T, \\
\Delta x = [\Delta x_1, \Delta x_2, \ldots, \Delta x_k]^T.
\]

Provided that the parameter changes are infinitesimal and the circuit function \( F \) is analytical at \( x^o \). We have a linear relation between the parameter changes and the resultant change in the single-circuit function \( F \). Thus, for the changes in the \( k \) parameters, we obtain the resultant change in \( F \) as follows:

\[
\Delta F = \sum_{j=1}^{k} \frac{\partial F}{\partial x_j} \Delta x_j, \tag{6a}
\]

or, in matrix notation:

\[
\Delta F = (\nabla F) \Delta x, \tag{6b}
\]

where the gradient operator is then given as

\[
\nabla = \left[ \frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \ldots, \frac{\partial}{\partial x_k} \right]. \tag{6c}
\]

If the parameter changes \( \Delta x_j \) are incremental, then the resultant change in a single-circuit function should be expressed as the Taylor series expansion including the \( 2^{nd} \)-order derivatives. Similar reasoning can be applied to the vector cases in eqs. (3), (4), and (5).

### III. SENSITIVITY ANALYSIS BY GAIN FACTORIZATION

#### A. Formulation

In the analysis of larger networks, it is often convenient to treat the whole network as a connection of smaller subnetworks and obtain an overall solution by combining individual solutions. One particular case, often occurring, is that of a cascaded network, as shown in Figure 1. The network consists of the inner (index \( I \)) two-port embedded between left (index \( L \)) and right (index \( R \)) two-ports. The two-port of index \( n \) is described by its chain matrix \( T_n \). Let us assume that our objective is to optimize gain performance \( G_T \) of the whole network in terms of some parameter \( x_n \) from the inner two-port. So, the overall transducer power gain \( G_T \) can be factorized into three terms as follows:

\[
G_T = \frac{P_L}{P_{AV_L}} = \frac{P_{AV_I}}{P_{AV_L}} \frac{P_{L_I}}{P_{L_I}}, \tag{7a}
\]

where all the power components have common definitions and are given in Figure 1. So,

\[
G_T = G_{AV_L} G_{T} G_{P_{L}}, \tag{7b}
\]

where \( G_{T} \) is the transducer power gain of the inner two-port, which is the function of the inner two-port parameters \( x_n, Z_{SI}, \) and \( Z_{LI} \) terminations; \( G_{AV_L} \) is the available power gain of the left-hand subnetwork, which does not depend upon any inner two-port’s parameter \( x_n \) and can be expressed as the chain parameters and source impedance of the left-hand subnetwork.
\[
G_{AL} = \prod_{i=1}^{n-1} G_{A_i} = \frac{R_S}{|A_L + C_L Z_0|^2 \text{Real} \left[ \frac{B_L + D_L Z_0}{A_L + C_L Z_0} \right]},
\]
\[(7c)\]

where
\[
T_L = \begin{bmatrix}
A_L & B_L \\
C_L & D_L
\end{bmatrix}
\]
\[
= \begin{bmatrix}
A_1 & B_1 \\
C_1 & D_1
\end{bmatrix} \cdots \begin{bmatrix}
A_{n-1} & B_{n-1} \\
C_{n-1} & D_{n-1}
\end{bmatrix}.
\]
\[(7d)\]

Similarly, \(G_{PR}\) in eq. (7b) is the operating power gain of the right-hand subnetwork, which consists of the final \(m-n\) cascaded two-ports and is independent from the inner-network parameters. It can be expressed in terms of the chain parameters and load of the right-hand subnetwork:

\[
G_{PR} = \prod_{i=n+1}^{m} G_{P_i} = \frac{R_L}{|D_R + C_R Z_0|^2 \text{Real} \left[ \frac{A_R Z_0 + B_R}{C_R Z_0 + D_R} \right]},
\]
\[(8a)\]

where
\[
T_R = \begin{bmatrix}
A_R & B_R \\
C_R & D_R
\end{bmatrix}
\]
\[
= \begin{bmatrix}
A_{n+1} & B_{n+1} \\
C_{n+1} & D_{n+1}
\end{bmatrix} \cdots \begin{bmatrix}
A_m & B_m \\
C_m & D_m
\end{bmatrix}.
\]
\[(8b)\]

Thus, using the equality in eq. (7b), the derivation of the overall transducer power gain with respect to the \(x_n\) parameter of the \(n^{th}\) two-port of \(m\) cascaded two-ports can be given as

\[
\frac{\partial G_T}{\partial x_n} = \frac{\partial G_{AN}}{\partial x_n} G_{PR},
\]
\[(9)\]

and gain sensitivity matrix of the \(m\)-cascaded network can be expressed as

\[
D_\Lambda(G_T) = [\nabla_{x_1}(G_T), \ldots, \nabla_{x_m}(G_T), \ldots, \nabla_{x_n}(G_T)],
\]
\[(10a)\]

where the gradient of the \(n^{th}\) two-port \(\nabla_{x_n}(G_T)\) can be given by

\[
\nabla_{x_n}(G_T) = \left[ \frac{\partial G_T}{\partial p_1}, \ldots, \frac{\partial G_T}{\partial p_{\ell}} \right]^T.
\]
\[(10b)\]

Here, the \(n^{th}\) port is assumed to have \(\ell\) sensitivity parameters.

**B. Formation of the Gain-Sensitivity Matrix for the IMC (T-Type) Transistor/OMC (T-Type) Circuit**

A microwave amplifier with a \(T\)-type distributed matching circuit can be considered as the cascaded connection of seven two-ports between the source \((V_S, Z_S)\) and the load \((Z_L)\), as shown in Figure 2. There are two typical two-ports used in the matching circuits whose sensitivities should be taken into account in the overall sensitivity matrix: (i) the series-line and (ii) the parallel-line two-ports, whose respective chain matrices can be given by

\[
T_S = \begin{bmatrix}
\cos \beta \ell & j Z_0 \sin \beta \ell \\
j \sin \beta \ell & \cos \beta \ell
\end{bmatrix},
\]
\[(11a)\]

Figure 2. \(T\)-type amplifier.
\[ T_p = \begin{bmatrix} 1 & 0 \\ \frac{j}{Z_\text{tan} \beta \ell} & 1 \end{bmatrix} \]  

where \( \ell \) and \( Z_0 \) are the physical length and the characteristic impedance for a line, respectively.

The transducer power gain of a two-port characterized by the chain parameters can be given by

\[ G_T = \frac{4R_SR_L}{A_{Z_\ell} + B + Z_\delta(Z_L + D)^2}, \]  

where \( Z_\delta = R_S + jX_S \) and \( Z_L = R_L + jX_L \) are the source and load impedances of the two-port, respectively.

Therefore, the gradient of a typical series-line two-port can be written as:

\[ \nabla_s(G_T) = \begin{bmatrix} \frac{\partial G_T}{\partial \ell_s} \\ \frac{\partial G_T}{\partial Z_{0s}} \end{bmatrix}^T, \]  

where

\[ \frac{\partial G_T}{\partial \ell_s} = -\frac{4R_SR_L}{P^2} \beta \left( S_2 + Z_{0s}^2 - S_1 \right) \sin 2\beta \ell_s 
+ \left( Z_{0s} S_4 - S_2 \right) \cos 2\beta \ell_s, \]  

\[ \frac{\partial G_T}{\partial Z_{0s}} = -\frac{4R_SR_L}{P^2} \left( S_2 - Z_{0s} \right) \sin^2 \beta \ell_s 
+ \left( S_4 + S_2 \right) \sin 2\beta \ell_s, \]  

\[ P = S_1 + \left( S_1 + Z_{0s}^2 - S_2 \right) \sin^2 \beta \ell_s 
+ \left( Z_{0s} S_4 - S_2 Z_{0s} \right) \sin 2\beta \ell_s, \]  

\[ S_1 = |Z_L|^2 + |Z_S|^2 + 2(R_SR_L + X_S X_L), \]  

\[ S_2 = |Z_L|^2 |Z_S|^2, \]  

\[ S_3 = 4X_S X_L + |Z_L|^2 + |Z_S|^2, \]  

\[ S_4 = X_L + X_S, \]  

\[ S_5 = X_S |Z_L|^2 + X_L |Z_S|^2. \]  

Similarly, the gradient vector for a typical parallel-line two-port can be expressed as

\[ \nabla_p(G_T) = \begin{bmatrix} \frac{\partial G_T}{\partial \ell_p} \\ \frac{\partial G_T}{\partial Z_{0p}} \end{bmatrix}^T, \]  

where

\[ \frac{\partial G_T}{\partial \ell_p} = \frac{4R_SR_L}{P^2} \frac{2\beta}{Z_{0p} \sin^2 \beta \ell_p} \left( R_2 Z_{0p} + T_3 \right), \]  

\[ \frac{\partial G_T}{\partial Z_{0p}} = \frac{4R_SR_L}{P^2} \frac{2}{Z_{0p} \sin^2 \beta \ell_p} \left( R_2 Z_{0p} + R_3 \right), \]  

\[ P = \frac{1}{Z_{0p} \sin^2 \beta \ell_p} \left( R_2 Z_{0p} + 2 T_3 \right), \]  

\[ T_1 = |Z_L|^2 + |Z_S|^2 + 2(R_SR_L + X_S X_L), \]  

\[ T_2 = \frac{|Z_L|^2 |Z_S|^2}{|Z_S|^2}, \]  

\[ T_3 = X_S |Z_L|^2 + X_L |Z_S|^2. \]  

Using these typical gradient vectors in the formation of the overall sensitivity matrix, the following points should be taken into consideration: (i) \( Z_S \) and \( Z_L \) are the input and output impedances of the left- and right-hand subnetworks, respectively, and (ii) \( G_{AV_{TL}} \) and \( G_{FR} \) should be placed as previously defined with respect to the inner two-port.

**IV SENSITIVITY ANALYSIS USING THE CHAIN-SENSITIVITY MATRIX**

**A. Formulation**

This method begins by obtaining the chain-sensitivity matrix of the overall network. The overall chain matrix is given by

\[ T = T_{TI} T_{TR}, \]  

where the sensitivity of the \( T \) matrix with respect to the \( x_n \) parameter of the \( n \)th port is given by

\[ \frac{\partial T}{\partial x_n} = T_L \frac{\partial T}{\partial x_n} T_R, \]  

where \( \frac{\partial T}{\partial x_n} \) is the chain-sensitivity matrix of the inner subnetwork with the sensitivity parameters.
The chain-sensitivity matrix is employed in the gain-sensitivity formula. For this purpose, the transducer gain can be expressed as

$$G_T = \frac{4R_L R_s}{P},$$  \hspace{1cm} (17a)

where

$$P = \Delta |AZ_L + B + Z_2(CZ_L + D)^2|$$ or

$$P = \Delta (AZ_L + B + Z_2(CZ_L + D))$$ \hspace{1cm} (17b)

The sensitivity of $G_T$ with respect to parameter $x_n$ can be written as

$$\frac{\partial G_T}{\partial x_n} = \sum_{i=1}^{4} \frac{\partial G_T}{\partial T_i} \frac{\partial T_i}{\partial x_n} + \sum_{i=1}^{4} \frac{\partial G_T}{\partial T_i^*} \frac{\partial T_i^*}{\partial x_n},$$ \hspace{1cm} (18a)

where $T_i$ stands for one of the overall chain parameters. The gain-sensitivity formula in eq. (18a) can be rewritten in matrix form as

$$\frac{\partial G_T}{\partial x_n} = \left[ \frac{\partial G_T}{\partial T_1} \frac{\partial T_1}{\partial x_n} + \frac{\partial G_T}{\partial T_2} \frac{\partial T_2}{\partial x_n} \right] + \left[ \frac{\partial G_T}{\partial T_3} \frac{\partial T_3^*}{\partial x_n} + \frac{\partial G_T}{\partial T_4} \frac{\partial T_4^*}{\partial x_n} \right],$$ \hspace{1cm} (18b)

where

$$\left[ \frac{\partial G_T}{\partial T_1} \right] = -\frac{4R_s R_L}{P^2} \left[ Z_2 M^* M^* Z_2 Z_2 M^* Z_2 M^* \right],$$ \hspace{1cm} (19a)

$$\left[ \frac{\partial G_T}{\partial T_2} \right] = -\frac{4R_s R_L}{P^2} \left[ Z_2^* M M Z_2 Z_2^* M Z_2^* M \right],$$ \hspace{1cm} (19b)

and because $4R_s R_L / P^2$ is real, we obtain

$$\left[ \frac{\partial G_T}{\partial T_3} \right] = \left[ \frac{\partial G_T}{\partial T_4} \right] = 0.$$

Figure 3. \hspace{1cm} II-type amplifier.

\[ \frac{\partial G_T}{\partial T_1} \] = \left[ \frac{\partial G_T}{\partial T_2} \right]^\ast, \hspace{1cm} (19c)

and the chain-sensitivity matrix can be defined as

$$\frac{\partial T_i}{\partial x_n} = \left[ \frac{\partial T_i}{\partial x_n} \right]^\ast,$$ \hspace{1cm} (20)

where we also obtain the following relation:

$$\frac{\partial T_i}{\partial x_n} = \left[ \frac{\partial T_i}{\partial x_n} \right]^\ast.$$ \hspace{1cm} (21)

B. Formation of Gain-Sensitivity Matrix for the IMC (II-Type) Transistor/OMC (II-Type) Circuit

Again we have seven cascaded two-ports between the source and the load in the II-type amplifier and the two typical matching circuits: (i) series-line and (ii) parallel-line two-ports (Fig. 3). Thus, the two typical chain-sensitivity matrices can be expressed as follows:

$$\left[ \begin{array}{cc} 0 & j \frac{Z_{os} \sin \beta \ell_p}{Z_{op} \tan \beta \ell_p} \\ j \frac{Z_{op} \sin \beta \ell_p}{Z_{os}} & 0 \end{array} \right],$$ \hspace{1cm} (22a)

$$\left[ \begin{array}{cc} 0 & j \frac{Z_{os} \sin \beta \ell_p}{Z_{op} \tan \beta \ell_p} \\ j \frac{Z_{op} \sin \beta \ell_p}{Z_{os}} & 0 \end{array} \right],$$ \hspace{1cm} (22b)

$$\left[ \begin{array}{cc} 0 & j \frac{Z_{os} \sin \beta \ell_p}{Z_{op} \tan \beta \ell_p} \\ j \frac{Z_{op} \sin \beta \ell_p}{Z_{os}} & 0 \end{array} \right],$$ \hspace{1cm} (22c)

$$\left[ \begin{array}{cc} 0 & j \frac{Z_{os} \sin \beta \ell_p}{Z_{op} \tan \beta \ell_p} \\ j \frac{Z_{op} \sin \beta \ell_p}{Z_{os}} & 0 \end{array} \right],$$ \hspace{1cm} (22d)
While using the above formulas, the order of the two-port in the chain should be taken into account by the relation given in eq. (16).

As an example, let us formulate the gain gradient $\nabla_{G_T}(G_T)$ of the fifth port in Figure 3:

$$\nabla_{G_T}(G_T) = \left[ \frac{\partial G_T}{\partial \ell_5} \frac{\partial G_T}{\partial Z_{05}} \right]$$

(23)

for the partial derivative with respect to $\ell_5$:

$$\frac{\partial G_T}{\partial \ell_5} = \left[ \frac{\partial G_T}{\partial T_{11}} \frac{\partial T_{11}}{\partial \ell_5} \right] + \left[ \frac{\partial G_T}{\partial T_{12}} \frac{\partial T_{12}}{\partial \ell_5} \right] \frac{\partial T_{12}^*}{\partial Z_{05}}$$

(24a)

$$\left[ \frac{\partial T_{11}}{\partial \ell_5} \right] = T_{11} T_{21}^* \frac{\partial T_{11}}{\partial Z_{05}} T_{6}$$

(24b)

$$\left[ \frac{\partial T_{12}}{\partial \ell_5} \right] = T_{12} T_{21}^* \frac{\partial T_{12}}{\partial Z_{05}} T_{6}$$

(24c)

$$\left[ T_{11c} \right] = \frac{\partial T_{11}}{\partial \ell_5}$$

(24d)

$$\left[ \frac{\partial G_T}{\partial Z_{05}} \right] = \frac{4R_R R_L}{P^2} [Z_L M^* M^* Z_L Z_L M^*] Z_L M^*]$$

(24e)

$$\left[ \frac{\partial T_{11}}{\partial Z_{05}} \right] = \left[ \begin{array}{ccc} T_{11c}(1, 1) & T_{11c}(1, 2) & T_{11c}(2, 1) & T_{11c}(2, 2) \\ T_{12c}(1, 1) & T_{12c}(1, 2) & T_{12c}(2, 1) & T_{12c}(2, 2) \end{array} \right]$$

(24f)

$$\left[ \frac{\partial T_{12}}{\partial Z_{05}} \right] = \left[ \begin{array}{ccc} T_{11c}(1, 1) & T_{11c}(1, 2) & T_{11c}(2, 1) & T_{11c}(2, 2) \\ T_{12c}(1, 1) & T_{12c}(1, 2) & T_{12c}(2, 1) & T_{12c}(2, 2) \end{array} \right]$$

(24g)

$$\left[ \frac{\partial T_{11}}{\partial Z_{05}} \right] = \left[ \begin{array}{ccc} \frac{\partial T_{11}}{\partial T_{11}} & \frac{\partial T_{11}}{\partial Z_{05}} \end{array} \right] \left[ \begin{array}{ccc} \frac{\partial T_{12}}{\partial T_{12}} & \frac{\partial T_{12}}{\partial Z_{05}} \end{array} \right]$$

(24h)

and for the partial derivative with respect to $Z_{05}$:

$$\frac{\partial G_T}{\partial Z_{05}} = \left[ \frac{\partial G_T}{\partial T_{11}} \frac{\partial T_{11}}{\partial Z_{05}} + \frac{\partial G_T}{\partial T_{12}} \frac{\partial T_{12}}{\partial Z_{05}} \right] \frac{\partial T_{12}^*}{\partial Z_{05}}$$

(25a)

$$\frac{\partial T_{11}}{\partial Z_{05}} = T_{11} T_{21}^* \frac{\partial T_{11}}{\partial Z_{05}} T_{6}$$

(25b)

$$\frac{\partial T_{12}}{\partial Z_{05}} = T_{12} T_{21}^* \frac{\partial T_{12}}{\partial Z_{05}} T_{6}$$

(25c)

$$\left[ \frac{\partial T_{11}}{\partial \ell_5} \right] = \left[ T_{11c}(1, 1) \ T_{11c}(1, 2) \ T_{11c}(2, 1) \ T_{11c}(2, 2) \right]^T$$

(25d)

$$\left[ \frac{\partial T_{12}}{\partial \ell_5} \right] = \left[ T_{12c}(1, 1) \ T_{12c}(1, 2) \ T_{12c}(2, 1) \ T_{12c}(2, 2) \right]^T$$

(25e)

$$\left[ \frac{\partial T_{11}}{\partial \ell_5} \right] = \left[ T_{11c}(1, 1) \ T_{11c}(1, 2) \ T_{11c}(2, 1) \ T_{11c}(2, 2) \right]^T$$

(25f)

$$\left[ \frac{\partial T_{12}}{\partial \ell_5} \right] = \left[ T_{12c}(1, 1) \ T_{12c}(1, 2) \ T_{12c}(2, 1) \ T_{12c}(2, 2) \right]^T$$

(25g)

V. COMPUTED RESULTS

A general computer program for the sensitivity analysis of the $T$ and $\Pi$ types of the distributed-parameter microwave amplifier was developed, based on the theory presented above. In addition, all the results of the analytical formulas were verified by the numerical perturbation method. The classical definition [eq. (2b)] of the sensitivity of the function with respect to parameter $x$ was used, which led to the following expressions in the perturbation-method case:

$$S_x = \frac{x \ F(x + \Delta x) - F(x)}{\Delta x}$$

(26)

it was assumed that $\Delta x = 10^{-4}$ cm and $10^{-4}$Ω

The validity of the techniques was tested on the $T$ and $\Pi$ types of the distributed-parameter circuits. Firstly, gain-sensitivity analyses of the IMC and OMC circuits of the $T$ and $\Pi$ distributed-parameter amplifiers are made and computed separately. In the analysis, it is assumed that the IMC circuits to be loaded by the complex conjugate of the transistor's required source impedance, $Z^*_{S_{TP}(\omega)}$, and the OMC
TABLE I. Physical Lengths and Characteristic Impedances for Matching Circuits

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Physical Lengths</th>
<th>Characteristic Impedances</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMC (T-Type)</td>
<td>( \ell_1 = 15.886 ) cm</td>
<td>( Z_{o1} = 36.310 \Omega )</td>
</tr>
<tr>
<td></td>
<td>( \ell_2 = 14.009 ) cm</td>
<td>( Z_{o2} = 180.885 \Omega )</td>
</tr>
<tr>
<td></td>
<td>( \ell_3 = 13.378 ) cm</td>
<td>( Z_{o3} = 53.041 \Omega )</td>
</tr>
<tr>
<td></td>
<td>( \ell_4 = 13.711 ) cm</td>
<td>( Z_{o4} = 129.006 \Omega )</td>
</tr>
<tr>
<td></td>
<td>( \ell_5 = 0.674 ) cm</td>
<td>( Z_{o5} = 166.347 \Omega )</td>
</tr>
<tr>
<td></td>
<td>( \ell_6 = 0.832 ) cm</td>
<td>( Z_{o6} = 95.109 \Omega )</td>
</tr>
<tr>
<td>OMC (T-Type)</td>
<td>( \ell_1 = 15.191 ) cm</td>
<td>( Z_{o1} = 199.988 \Omega )</td>
</tr>
<tr>
<td></td>
<td>( \ell_2 = 13.678 ) cm</td>
<td>( Z_{o2} = 114.411 \Omega )</td>
</tr>
<tr>
<td></td>
<td>( \ell_3 = 15.197 ) cm</td>
<td>( Z_{o3} = 193.972 \Omega )</td>
</tr>
</tbody>
</table>

As far as the developed theory is concerned, a simple method called the chain-sensitivity matrix has been suggested to formulate the gain sensitivities with respect to the circuit parameters of the cascaded two-ports due to the following reasons: (i) this method is relatively very simple to form the derivative vectors of \([\partial G/\partial T]\) and \([\partial T/\partial x_n]\), the chain-sensitivity matrix method is preferable to the gain-factorization method. A clear advantage of the analytical methods can be found in [5], where a comparison of the CPU time with respect to the number of variables for the two methods is graphically presented.

VI. CONCLUSION

The significance of this work can be discussed in two aspects: (i) the contributions of the sensitivity analysis developed for the cascaded two-ports to the circuit theory and (ii) the importance of the problem focused upon in this work, with regard to applications.

As far as the developed theory is concerned, a simple method called the chain-sensitivity matrix has been suggested to formulate the gain sensitivities with respect to the circuit parameters of the cascaded two-ports due to the following reasons: (i) this method is relatively simple, as compared to the gain factorization method in the literature [4]; (ii) it is easily extendable to the sensitivities of the other performance measure functions, such as noise figure, input VSWR, and output VSWR; (iii) its fundamentals are also to extendable to other types of configurations.

As far as applications are concerned, the significance of the work can be summarized briefly as follows. Efficient, gradient-based optimization techniques require calculation of the circuit-performance sensitivities. In this work, analytical formulas for direct computation are presented in a form of compatible with the analysis of the given circuit. These correlations to each other. The design of these amplifiers was originally presented by Günes and Cengiz in [10]. The design targets of the noise, input VSWR, and gain for both types of amplifiers were \( F_{req} = 0.46 \) dB (\( \Leftrightarrow 1, 12 \)) and \( V_{freq} = 1 \), \( G_{freq} = 12 \) dB (\( \Leftrightarrow 15, 86 \)), respectively, in the frequency range 2–11 GHz. NE329S01 was used as the active element. The nominal values of the transmission-line segments used in the matching circuits are given in Table I and the corresponding gain-frequency characteristics of the \( T \) and \( \Pi \) types of circuits are given in Figures 4(a) and 4(b), and Tables II and III, respectively.

The desired behavior of the sensitivity of an element can be defined as the magnitude of its normalized sensitivity to take values between zero and unity, as in eq. (2d):

\[
0 < |S_s(F)| < 1.
\]

The sensitivity-frequency curves of the \( T \) and \( \Pi \) types of amplifiers with respect to the parameters of the matching circuits are given in Figures 5–16, in comparison to each other. As seen from the figures, gain can be considered not sensitive to the physical lengths and characteristic impedances of the lines in both configurations along the operation bandwidth, except the sensitivity with respect to the physical lengths and characteristic impedances \( \ell_5, Z_{os} \) of the fifth element in the \( \Pi \) configuration. Thus, the \( T \)-type amplifier is preferable to the \( \Pi \)-type amplifier. However, as it is relatively very simple to form the derivative vectors of \([\partial G/\partial T]\) and \([\partial T/\partial x_n]\), the chain-sensitivity matrix method is preferable to the gain-factorization method.

Figure 4. Gain-frequency characteristic of the (a) \( T \)-type amplifier and (b) \( \Pi \)-type amplifiers.
TABLE II. Gain-frequency Characteristics of the T-Type Amplifier

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_T$ (ratio)</td>
<td>11.6</td>
<td>14.8</td>
<td>14.5</td>
<td>13.6</td>
<td>13.5</td>
<td>12.6</td>
<td>12.4</td>
<td>12.5</td>
<td>11.7</td>
<td>10.1</td>
<td>8.0</td>
<td>5.8</td>
</tr>
</tbody>
</table>

TABLE III. Gain-frequency Characteristics of the H-Type Amplifier

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_T$ (ratio)</td>
<td>19.6</td>
<td>20.9</td>
<td>18.7</td>
<td>17.1</td>
<td>17.7</td>
<td>17.5</td>
<td>18.4</td>
<td>19.8</td>
<td>20.5</td>
<td>19.4</td>
<td>16.2</td>
<td>12.4</td>
</tr>
</tbody>
</table>

Figure 5. Sensitivity-frequency curves of the T-type amplifier with (a) $\ell_1$ variation and (b) $Z_{o1}$ variation. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com].

Figure 6. Sensitivity-frequency curves of the H-type amplifier with (a) $\ell_1$ variation and (b) $Z_{o1}$ variation. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com].

Figure 7. Sensitivity-frequency curves of the T-type amplifier with (a) $\ell_2$ variation and (b) $Z_{o2}$ variation. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com].
Figure 8. Sensitivity-frequency curves of the Π-type amplifier with (a) $\ell_2$ variation and (b) $Z_{o2}$ variation. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com].

Figure 9. (a) Sensitivity-frequency curve of the $T$-type amplifier with $\ell_3$ variation and (b) $Z_{o3}$ variation. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com].

Figure 10. Sensitivity-frequency curves of the Π-type amplifier with (a) $\ell_3$ variation and (b) $Z_{o3}$ variation. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com].

Figure 11. Sensitivity-frequency curve of the $T$-type amplifier with (a) $\ell_4$ variation and (b) $Z_{o4}$ variation. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com].
Figure 12. Sensitivity-frequency curves of the II-type amplifier with (a) \( \ell_s \) variation and (b) \( Z_{os} \) variation. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com].

Figure 13. Sensitivity-frequency curve of the \( T \)-type amplifier with (a) \( \ell_s \) variation and (b) \( Z_{os} \) variation. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com].

Figure 14. Sensitivity-frequency curves of the II-type amplifier with (a) \( \ell_s \) variation and (b) \( Z_{os} \) variation [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com].

Figure 15. Sensitivity-frequency curve of the \( T \)-type amplifier with (a) \( \ell_s \) variation and (b) \( Z_{os} \) variation. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]
sensitivities can be related to any passive or active parameter of the network. In the CAD of amplifiers with high gain, the direct expressions for gain sensitivities improve the speed of computations, and also safeguard against possible numerical difficulties associated with the perturbation method.

The other important application area with regard to sensitivity analysis, without a doubt, is tolerance analysis. Since sensitivity can be used to predict the response variations due to changes in the values of the circuit elements for various reasons, this analytical work is expected to be a good reference for the computer-aided optimal-tolerance assignment, optimal centering, and postproduction tuning.

REFERENCES

BIOGRAPHIES

Filiz Güneş received her M.S. degree in electronic and communication engineering from the Istanbul Technical University in 1972. She attained her PhD. degree in communication engineering from Bradford University, London, in 1979. She worked as a research fellow at the same university between 1979 and 1983 with contracts from the European Space Agency and British Defence Minister in the areas of the propagation and electromagnetic compatibility. Since 1983, she has been with the Yıldız Technical University, where she is currently a Full Professor and head of the Electronic and Communication Engineering Department. Her current research interests are in the areas of multivariable network theory, device modeling, computer-aided circuit design, microwave amplifiers, microwave filters, broadband matching circuits, monolithic microwave integrated circuits, and electromagnetic compatibility.

Serhat Altunç was born in Turkey. He earned an undergraduate degree in Electronics and Communications Engineering. After receiving his M. Engg. Degree from Istanbul Technical University, he continued his graduate study at Yıldız Technical University receiving a MS (Communication Engineering, 2003). He worked as a Research Assistant at Yıldız Technical University from 2001 to 2003. He has published two papers.